MEASUREMENT OF THE THERMAL RESISTANCE AND THERMAL RESPONSE OF DIFFUSED SILICON TRANSISTORS

By H. B. Grutchfield

I. INTRODUCTION

The conventional ratings for power dissipation and junction temperature assigned to transistors apply to continuous operation but are not applicable to power pulse conditions. The main problem is that the definition of power pulse conditions for transistors cannot be generalized because each type of material and structure has a unique thermal characteristic.

Operation of diffused silicon transistors at junction temperatures in excess of their maximum rating might not be immediately apparent because sudden failure seldom occurs. However continued operation of diffused silicon devices over rating can cause a gradual deterioration of certain critical parameters such as h_{FE} and I_{CBO}.* This may lead to catastrophic failure such as produced by thermal run away, or parameter deterioration to the point of circuit inoperation. Also it is permissible to operate diffused silicon transistors over the continuous maximum power rating for short pulse periods such as in nonsaturating switching circuits. Therefore a knowledge of the thermal response characteristics of the transistor can be very desirable.

This paper describes a simple method of determining the thermal resistance and the thermal response of a transistor.

II. SEVERAL METHODS OF MEASURING THE THERMAL RESISTANCE OF A TRANSISTOR

There are several possible methods of measuring a transistor's thermal resistance, all of which utilize one of the temperature sensitive parameters as a thermometer.

One such parameter is the collector-base cutoff current, I_{CBO}. It is satisfactory as a temperature gauge for germanium transistors, but for silicon transistors, especially of the planar construction, leakage current is so low that the desired accuracy is difficult to obtain. Furthermore, the resistive leakage can swamp out diode leakage thus masking the I_{CBO} to temperature relationship.

The forward voltage drop of the collector-base junction, V_{BC}, is another temperature sensitive parameter. However, thermal measurements with V_{BC} include a problem which is particularly severe with small geometry transistors. Since the collector-base junction is reverse biased during the ON or power dissipating condition, it is necessary to forward bias the junction before applying measuring current. Small geometry devices have very short thermal time constants and, unless the switching interval is quite short, a significant degree of cooling can occur during the switching interval. Also, the bulk resistivity has a positive temperature coefficient, while the forward biased diode has a negative temperature coefficient. For this reason, the use of V_{BC} as a temperature sensitive parameter may be complicated by the effects of the collector bulk resistivity. This effect can be minimized if the measuring current is small.

The preferred temperature sensitive parameter for measuring the thermal resistance of diffused silicon transistors is the forward voltage drop of the base-emitter junction, V_{BE}. The parameter V_{BE} is relatively easy to measure and the results obtained are reproducible.

*Operation over ratings of some of the older alloy and grown-junction transistors was prohibitive for even short periods because of the low temperature solders used in their construction.

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III. MEASURING THERMAL RESISTANCE UTILIZING V_{BE}

The V_{BE} thermometer is used to monitor the change in the junction temperature as a function of the power dissipated. The measurement circuit is illustrated in Fig. 1. Mercury relay S_1 switches from high- to low-power dissipation. The circuit is useful up to the maximum current rating of this relay. A transistor switch could be substituted; however, a transistor that is fast enough to achieve the desired results will limit the maximum current to less than one ampere.

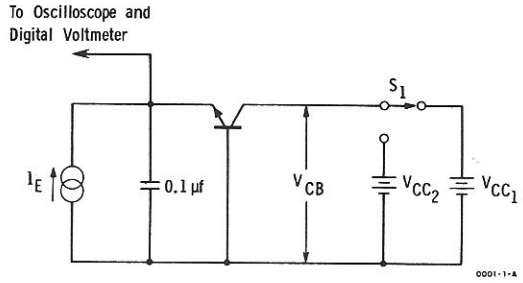


Fig. 1. Thermal resistance measurement circuit

The emitter current, I_E, must be constant during all measurements. Therefore, either a constant current supply or a large emitter dropping resistor should be used such that V_{EE} is about 100 times V_{BE}. The use of a power decade-resistance box will provide the needed versatility for a wide current range.

The circuit shown provides maximum versatility with the two collector supplies. One supply and two zener diodes can be substituted, but versatility will be greatly curtailed.

A linear relationship exists between V_{BE} and temperature. Since I_E is constant for all measurements, power dissipation is proportional to the collector voltage, V_{CB}.

The transistor's thermal characteristics can be measured in any type of environment. For an infinite heat sink or the junction-to-case value, a heat dissipating bath is used to supply a stable temperature reference point.

The measurement steps are as follows:

A. JUNCTION-TO-CASE THERMAL RESISTANCE (θ_{J-C})

1) Calibrate the V_{BE} Thermometer

Place transistor in an agitated silicone oil bath containing a heater element with external control, and set transistor bias to a selected low power dissipation. Measure V_{BE} at several discrete points with increasing temperature. (A high impedance digital voltmeter with three or four digit readout is recommended for easy and accurate readings.) Next a curve is plotted from these readings. Its slope will be about 1.5 to 2.2 mV/°C for diffused silicon transistors within the normal operating current range (see Fig. 2).

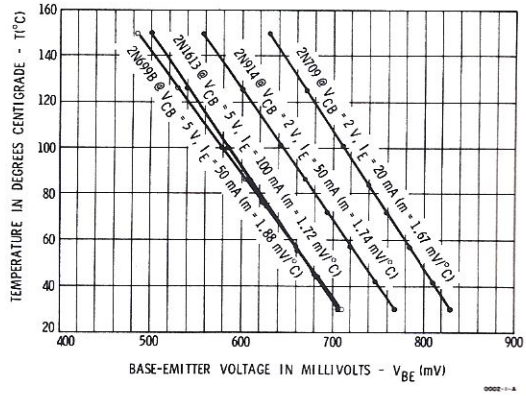

TEMPERATURE IN DEGREES CENTIGRADE - Tj(°C)
160
140
120
100
80
60
40
20
400
500
600
700
800
900
BASE-EMITTER VOLTAGE IN MILLIVOLTS - V_{BE} (mV)
2N698 @ V_{CB} = 5 V, I_E = 50 mA (m = 1.88 mV/°C)
2N1613 @ V_{CB} = 5 V, I_E = 100 mA (m = 1.72 mV/°C)
2N914 @ V_{CB} = 2 V, I_E = 50 mA (m = 1.74 mV/°C)
2N709 @ V_{CB} = 2 V, I_E = 20 mA (m = 1.67 mV/°C)

Fig. 2. V_{BE} vs. temperature for several typical silicon-planar transistors

2) Determine Case-to-Bath Thermal Resistance

The case-to-bath thermal resistance can be determined in one of two ways: either empirically or by an analysis of the thermal networks associated with the transistor. Derivation of the networks are discussed in Section IV.

The empirical method is as follows: Attach a thermocouple to transistor case as close to the die as possible. Next, place device in an agitated bath. Although acetone is preferred, silicone oil can be used. However it has a higher thermal resistance between the transistor case and bath. Measure the temperature rise when power is applied and calculate the thermal resistance value.

3) Measure V_{BE} as a Function of Power Dissipation

First measure V_{BE} in a selected high power condition, and second in the previously selected low power condition. The unit should be allowed to stabilize at each condition. The internal feedback voltage, V_{12}, produced by the change in the collector voltage, can be measured on an oscilloscope with a high-gain differential preamplifier. The horizontal sweep is set at about 100 microseconds (10 μsec/cm). For very small geometry devices, a sweep speed of about 20 microseconds (2 μsec/cm) may be required. The display will be as shown in Fig. 3 and V_{12} is equal to $V_{BE_1} - V_{BE_2}$.

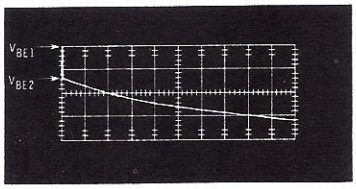


Fig. 3. Oscilloscope display for measurement of V_{12}

4) Calculate Thermal Resistance

Calculate transistor thermal resistance as follows:

$$\theta_{J-C} = \frac{\left(V_{BE_3} - V_{BE_1} - V_{12}\right)/\Delta P}{\Delta V_{BE}/\Delta T} - \theta_{C-B} \qquad (1)$$

where

V_{BE_3} = V_{BE} at low power
V_{BE_1} = V_{BE} at high power
$\Delta V_{BE}/\Delta T$ = slope of V_{BE} versus temperature
θ_{C-B} = thermal resistance between the transistor case and the bath

B. JUNCTION-TO-AMBIENT THERMAL RESISTANCE (θ_{J-A})

The thermal resistance, junction-to-ambient, θ_{J-A}, is determined in the same manner as θ_{J-C} following procedures 1 to 4 above, with the exception that the transistor is placed in an environment of free air. The testing area must be free of any appreciable air currents, in order to obtain a valid measurement. The device is shielded by placing it in a box with dimensions slightly greater than one cubic foot.

If θ_{J-C} is a known parameter or is required for analysis, an alternate method of determining θ_{J-A} can be used; i.e., measure θ_{C-A} in the free air environment. Then θ_{J-A} can be calculated since

$$\theta_{J-A} = \theta_{J-C} + \theta_{C-A} \qquad (2)$$

Precautions in Measurements

There are several second order effects which may introduce significant errors unless suitable precautions are observed during measurements.

1) As I_E increases the slope of V_{BE} versus temperature decreases due to the positive temperature coefficient of the bulk resistivity and the IR drop in the base region. Figure 4 shows the relationship between the slope of $\Delta V_{BE}/\Delta T$ and I_E for a typical 2N1613 transistor.

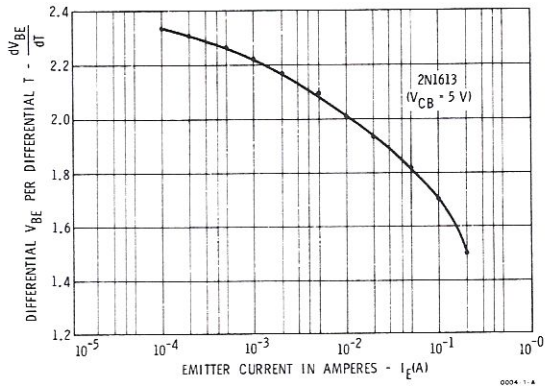


Fig. 4. Characteristic curve of $\Delta V_{BE}/\Delta T$ vs. I_E for type 2N1613 transistor

2) Base current, I_B, increases significantly as the unit approaches saturation, and the IR drop within the base region increases external V_{BE}.

3) If the selected V_{CB} is too low the unit may approach saturation as the bath temperature increases because the collector-to-emitter voltage has a positive temperature coefficient. This is observed in Fig. 5 as a decreasing slope. At I_E = 200 mA, the test unit saturates at a bath temperature of 25°C.

4) At very high temperatures I_{CBO} may become appreciable, and the measured slope begins to increase. This is also shown in Fig. 5.

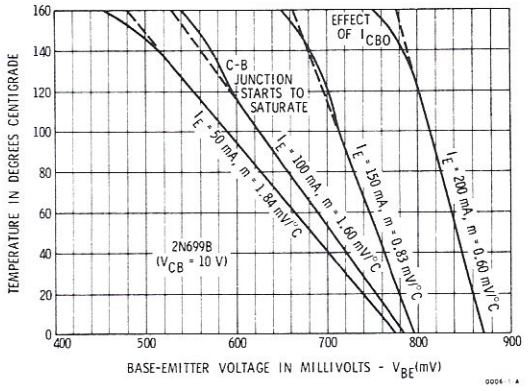


Fig. 5. Characteristic curves of V_{BE} vs. temperature for type 2N699B transistor

5) The parameter V_{CB} in the high power condition should not exceed one-half LV_{CEO} for best results. However, a few very-fast switching types require a V_{CB} near the LV_{CEO} value in the high power condition to obtain a reasonable ΔP. This is because the internal feedback voltage, V_{12}, is proportional to ΔV_{CB} and, to a lesser degree, I_E. When V_{12} becomes large with respect to ΔV_{BE}, the desired accuracy is difficult to obtain.

6) High power V_{CB} should not exceed LV_{CEO}. Not only does the high V_{12} affect the accuracy of the measurement, but as V_{CB} approaches LV_{CEO}, the common base forward current transfer ratio, α, approaches unity. At a V_{CB} greater than LV_{CEO}, α is greater than unity. Since by definition α is equal to I_C/I_E, current is pulled out of the base. Not only is the external V_{BE} measurement effected, but also the high current density at the collector-base junction, due to a "pinching-in" of the collector current by the transverse base current, may produce localized hot spots.

IV. MEASURING THE THERMAL RESPONSE OF A TRANSISTOR

An examination of the cooling curve of a diffused silicon transistor, Fig. 7A, reveals the distributed nature of the thermal resistances of the various parts. However, a thermal equivalent circuit can be derived, and the values of the lumped parameters determined, by methods developed by Strickland.[1] The transistor thermal response is found by an analysis of the thermal equivalent circuit.

The Thermal Equivalent Circuit

It has been shown[1] that the thermal equivalent circuit of a transistor can be represented as an infinite string of parallel RC networks, where R is the electrical analogue of thermal resistance and C is the electrical analogue of thermal capacitance. The total temperature drop is the sum of the temperature drops across each thermal RC network.

$$T_j(t) = \sum_{n=1}^{\infty} T_n(t) \qquad (3)$$

Since it is a necessary condition that the junction temperature, T_j, have a finite value, the solution must be convergent and the thermal equivalent circuit can be represented by a finite number of RC networks (Fig. 6).

$$T_j(t) = \sum_{n=1}^{m} T_n(t) = \sum_{n=1}^{m} T_n(0)\epsilon^{-t/R_n C_n} \qquad (4)$$

where

$$T_n(0) = T_n(t) \quad \text{at} \quad T = 0$$

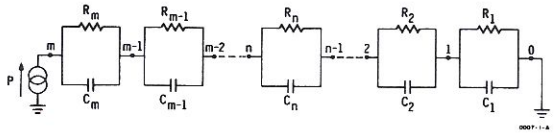


Fig. 6. Thermal equivalent circuit

The value of m is determined by independent measurement of thermal resistance and the desired degree of accuracy.

1) Determination of Thermal Network Values

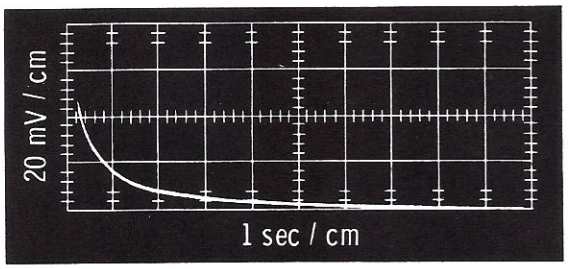
The slope of $\Delta V_{BE}/\Delta T$ and the thermal resistance in the desired environment (acetone in the case of the following example) must be found prior to determining the parameter values of the thermal equivalent circuit.

The V_{BE} thermometer is employed to measure the change in T_j vs. time. A series of oscillograms of V_{BE} vs. time are taken from an oscilloscope having a high-gain differential preamplifier (same as in Sec. III A, Step 3).

The transistor bias conditions for the example shown were: I_E = 100 mA, V_{CB_1} = 15 V, V_{CB_2} = 5 V, and ΔP = 1.0 W. For the required curve plots, the results of the independent measurements from the methods of Sec. III for type 2N1613 are used: $\Delta V_{BE}/\Delta T$ = 1.75 mV/°C and θ_{J-B} = 50.2°C/W.

First, an oscillogram of the total change of V_{BE} is taken, Fig. 7A, which is then plotted on semilog paper as T_j vs. time, Fig. 8A. The plot approaches a straight line and this portion is assumed to be the contribution of the exponential term with the longest time constant. The straight line is extrapolated to time zero. The difference between the cooling curve and the extrapolated straight line is assumed to be the contribution of the remaining exponential terms. A second oscillogram is taken, Fig. 7B, which includes this difference; and again the cooling curve is plotted, Fig. 8B. This procedure is continued until a curve is obtained in which the difference between the cooling curve and the extrapolated straight line is negligible, Fig. 8E.

A



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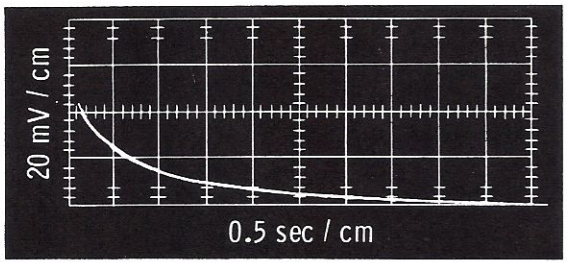

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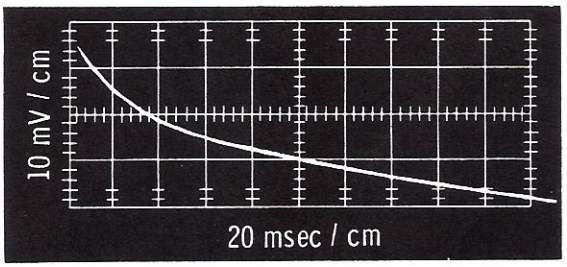

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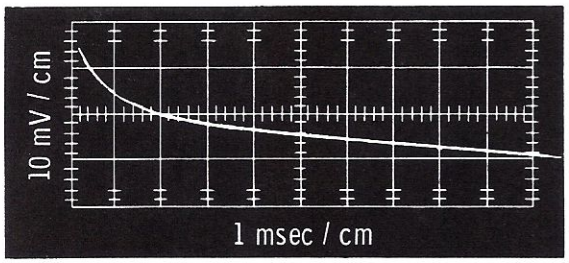


E

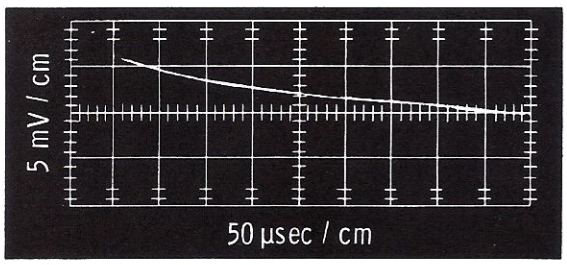
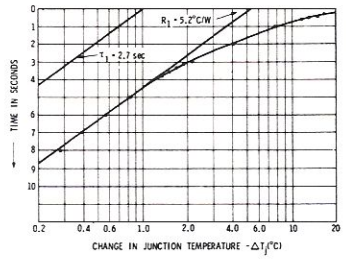
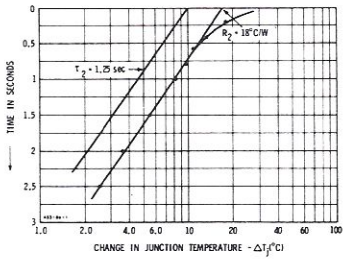
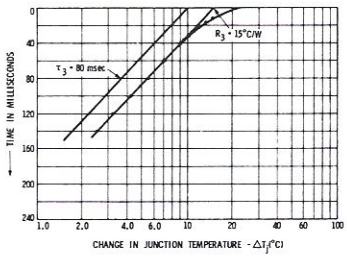
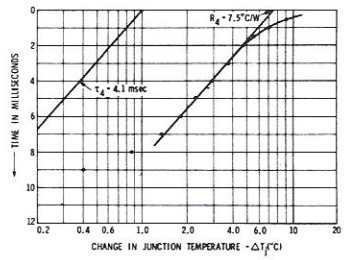


Fig. 7. Oscillograms of V_{BE} vs. time for type 2N1613 test unit









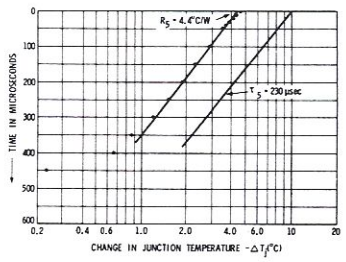


Fig. 8. Graphical analysis of cooling curve of type 2N1613 test unit

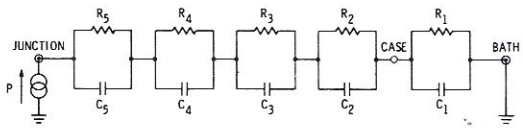
2) Calculations From the Curve Data

The values of thermal resistance for each network are:

$$R = \frac{\Delta T_j}{\Delta P} \tag{5}$$

The time constant for each network is also taken from the curves. The first network is assumed to be the thermal resistance between the transistor case and the bath. This has been confirmed by independent measurements.

The parameter values can now be added to the thermal equivalent circuit* (Fig. 9).



$R_1 = 5.2°C/W$ $C_1 = 520 \times 10^{-3}$ joule/°C $\tau_1 = 2.7$ sec
$R_2 = 18°C/W$ $C_2 = 69 \times 10^{-3}$ joule/°C $\tau_2 = 1.25$ sec
$R_3 = 15°C/W$ $C_3 = 5.3 \times 10^{-3}$ joule/°C $\tau_3 = 80 \times 10^{-3}$ sec
$R_4 = 7.5°C/W$ $C_4 = 547 \times 10^{-6}$ joule/°C $\tau_4 = 4.1 \times 10^{-3}$ sec
$R_5 = 4.4°C/W$ $C_5 = 52 \times 10^{-6}$ joule/°C $\tau_5 = 230 \times 10^{-6}$ sec

0013-1-A

Fig. 9. Thermal equivalent circuit of 2N1613 test unit

Applications of the Thermal Equivalent Circuit

One of the obvious applications of the thermal equivalent circuit is in switching applications where periodic rectangular power pulses, of duration t_1, are applied to the transistor at intervals of t_p (Fig. 10). An analysis of the thermal equivalent circuit will yield an equation for the junction temperature rise at any time, t. This is done in the Appendix for periodic rectangular pulses. A similar analysis is possible for any other waveform.

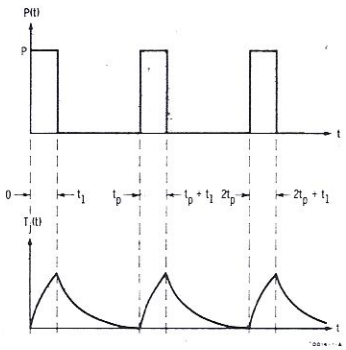


Fig. 10. The steady-state junction temperature response to periodic rectangular power pulses

Adding a constant term to include the ambient temperature, T_A, to equations (27-A) and (28-A) of the Appendix, allows us to compute the maximum and minimum junction temperatures.

$$T_j(max) = P \sum_{n=1}^{m} R_n \left[\frac{1 - \epsilon^{-(t_1/\tau_n)}}{1 - \epsilon^{-(t_p/\tau_n)}} \right] + T_A \tag{6}$$

$$T_j(min) = P \sum_{n=1}^{m} R_n \epsilon^{-(t_p - t_1/\tau_n)} \left[\frac{1 - \epsilon^{-(t_1/\tau_n)}}{1 - \epsilon^{-(t_p/\tau_n)}} \right] + T_A \tag{7}$$

For a very short duty cycle (t_p approaches infinity), then

$$T_j(max) = P \sum_{n=1}^{m} R_n \left[1 - \epsilon^{-(t_1/\tau_n)} \right] + T_A \tag{8}$$

and

$$T_j(min) = T_A \tag{9}$$

Using equation (6) curves of peak power versus pulse width at a constant duty cycle can be plotted.

V. CONCLUSION

A method of measuring the thermal resistance of a diffused silicon transistor has been discussed showing that the most desirable method utilizes V_{BE} as a thermometer. Next a thermal equivalent circuit, represented by a finite string of RC networks, was derived from which the transistor's thermal response can be analyzed.

The detailed thermal response characteristics are quite valuable in designing power pulse applications, more so than continuous operating conditions given in transistor data sheets.

Expressions for the steady-state maximum and minimum junction temperatures for power pulse conditions are derived in the Appendix.

*It should be recognized that the derived thermal equivalent circuit is not valid for avalanche switching.[2] However, a valid thermal equivalent circuit for a transistor in avalanche breakdown can be derived by methods identical to those discussed in this paper.

APPENDIX

Equations for maximum and minimum junction temperatures of a transistor for the case of periodic excitations can be derived by careful analysis of its thermal equivalent circuit. The temperature variation across only a single network need be analyzed since the other networks will be identical in form, differing only in the values of R and C. The total, or junction, temperature response can be obtained by summing all of the networks.

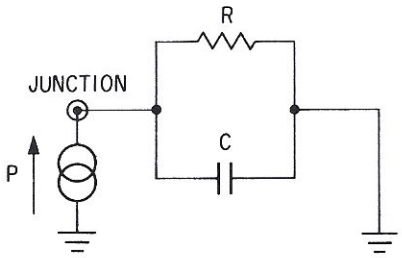


Fig. 1A. A single R-C network

Consider the circuit of Fig. 1-A for the initial case of no applied power. The node equation for a given P(t) is

$$\frac{T(t)}{R} + C\,\frac{dT(t)}{dt} = P(t) \qquad (1\text{-A})$$

where P is the applied power and T is the junction temperature.

Application of Laplace transformations yields,

$$\frac{T(s)}{R} + \left[Cs\,T(s) + T(0+)\right] = P(s)$$

For the case of initially no applied power, $T(0+) = 0$, therefore

$$T(s) = \frac{RP(s)}{RCs + 1}$$

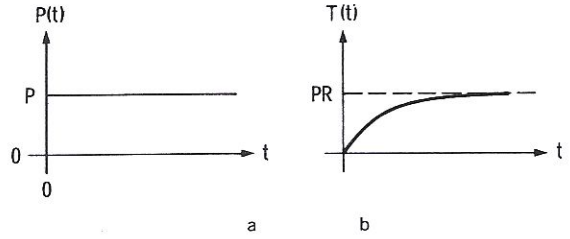


Fig. 2A. (a) Unit step function of power
(b) Response of single network

1. First, consider the case where P(t) is a unit step function, Fig. 2-A(a). The solution of this equation is

$$T(t) = PR\left(1 - \varepsilon^{-t/RC}\right) \qquad (2\text{-A})$$

Substituting $\tau = RC$, we have

$$T(t) = PR\left(1 - \varepsilon^{-t/\tau}\right) \qquad (3\text{-A})$$

This is plotted in Fig. 2-A(b).

$$\lim_{t\to\infty} T(t) = PR \qquad (4\text{-A})$$

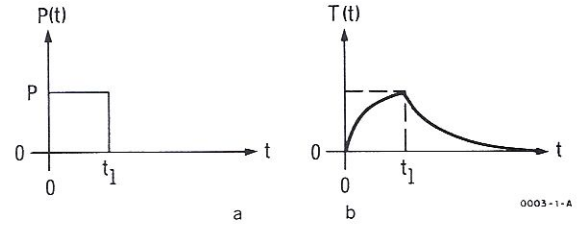


Fig. 3A. (a) Single rectangular power pulse
(b) Response of single network

2. Next, consider the case for a single rectangular power pulse of duration t_1, Fig. 3-A(a). In this case

$$P(s) = \frac{P}{s}\left(1 - \varepsilon^{-t_1 s}\right)$$

$$T(s) = \frac{PR\left(1 - \varepsilon^{-t_1 s}\right)}{s(RCs + 1)}$$

The solution is

$$T(t) = PR\left(1 - \varepsilon^{-t/\tau}\right) \text{ for } 0 \le t \le t_1 \qquad (5\text{-A})$$

$$T(t) = PR\left[\varepsilon^{-(t-t_1)/\tau}\left(1 - \varepsilon^{-t_1/\tau}\right)\right]$$

$$\text{for } t \ge t_1 \qquad (6\text{-A})$$

The exact shape of T(t) is dependent upon the relative values, but the plot will be of the form shown in Fig. 3-A(b).

$$\lim_{t\to o} T(t) = \lim_{t\to o} \left[PR\left(1 - \varepsilon^{-t/\tau}\right)\right] = 0 \qquad (7\text{-A})$$

$$\lim_{t\to t_1} T(t) = \lim_{t\to t_1} \left[PR\left(1 - \varepsilon^{-t/\tau}\right)\right]$$

$$= \lim_{t\to t_1} \left\{PR\left[\varepsilon^{-(t-t_1)/\tau}\left(1 - \varepsilon^{-t_1/\tau}\right)\right]\right\}$$

$$= PR\left(1 - \varepsilon^{-t_1/\tau}\right) \qquad (8\text{-A})$$

$$\lim_{t\to \infty} T(t) = \lim_{t\to \infty} \left[\varepsilon^{-(t-t_1)/\tau}\left(1 - \varepsilon^{-t/\tau}\right)\right] = 0 \qquad (9\text{-A})$$

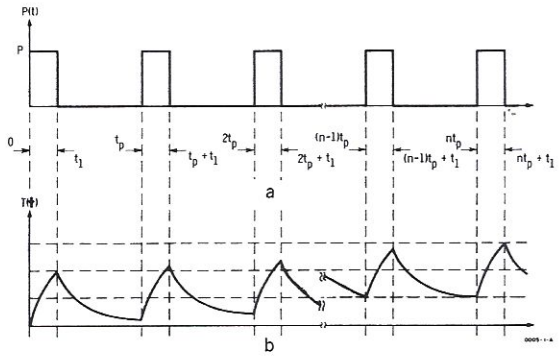


Fig. 4A. (a) Periodic power pulses
(b) Response of single network

3. Finally we will consider the case of periodic rectangular power pulses of duration t_1. The time of one period is t_p, Fig. 4-A(a). In this case

$$P(s) = \frac{P}{s}\left(\frac{1 - \varepsilon^{-t_1 s}}{1 - \varepsilon^{-t_p s}}\right)$$

$$T(s) = \frac{PR}{s(RCs + 1)}\left(\frac{1 - \varepsilon^{-t_1 s}}{1 - \varepsilon^{-t_p s}}\right)$$

$$T(t) = \mathcal{L}^{-1}\left[\frac{PR}{s(RCs + 1)}\left(\frac{1 - \varepsilon^{-t_1 s}}{1 - \varepsilon^{-t_p s}}\right)\right]$$

$$= PR\left\{\left[\left(1 - \varepsilon^{-t/\tau}\right)U(t) - \left(1 - \varepsilon^{-(t-t_1)/\tau}\right)U(t - t_1)\right] \cdot \left[1 + \varepsilon^{t_p/\tau} + \varepsilon^{2t_p/\tau} + \cdots + \varepsilon^{\frac{(n-1)t_p}{\tau}}\right]\right\}$$

By geometric progression:

$$1 + x + x^2 + \cdots + x^{n-1} = \sum_{k=0}^{n-1} x^k = \frac{x^n - 1}{x - 1}$$

and

$$T(t) = PR\left[\left(1 - \varepsilon^{-t/\tau}\right)\left(\frac{\varepsilon^{nt_p/\tau} - 1}{\varepsilon^{t_p/\tau} - 1}\right)\right] \qquad (10\text{-A})$$

for

$$(n - 1)t_p \le t \le (n - 1)t_p + t_1$$

$$T(t) = PR\left[\varepsilon^{-(t-t_1)/\tau}\left(1 - \varepsilon^{-t_1/\tau}\right)\left(\frac{\varepsilon^{nt_p/\tau} - 1}{\varepsilon^{t_p/\tau} - 1}\right)\right] \qquad (11\text{-A})$$

for

$$(n - 1)t_p + t_1 \le t \le nt_p$$

The general shape of T(t) is plotted in Fig. 4-A(b).

With the aid of equations (10-A) and (11-A) we can determine T(max) and T(min).

a. First, for the case where $(n - 1)t_p \le t \le (n - 1)t_p + t_1$:

$$T(t) = PR\left[\left(1 - \varepsilon^{-t/\tau}\right)\left(\frac{\varepsilon^{nt_p/\tau} - 1}{\varepsilon^{t_p/\tau} - 1}\right)\right] \qquad (10\text{-A})$$

Rearranging terms,

$$[T(t)]_{transient} = PR\left(\frac{\varepsilon^{-t/\tau} - 1}{\varepsilon^{t_p/\tau} - 1}\right)$$

$$= PR\left[\varepsilon^{-t/\tau}\left(\frac{1 - \varepsilon^{t/\tau}}{\varepsilon^{t_p/\tau} - 1}\right)\right] \qquad (12\text{-A})$$

$$\lim_{t\to o}[T(t)]_{transient} = \lim_{t\to o}\left\{PR\left[\varepsilon^{-t/\tau}\left(\frac{1 - \varepsilon^{t/\tau}}{\varepsilon^{t_p/\tau} - 1}\right)\right]\right\}$$

$$= 0 \qquad (13\text{-A})$$

$$[T(t)]_{\text{steady state}} = PR\left[\frac{\varepsilon^{nt_p/\tau} - \varepsilon^{-(t-nt_p)/\tau}}{\varepsilon^{t_p/\tau} - 1}\right]$$

$$= PR\left[\frac{\varepsilon^{(n-1)t_p/\tau}\left(1 - \varepsilon^{-t/\tau}\right)}{1 - \varepsilon^{-t_p/\tau}}\right] \quad (14\text{-A})$$

Noting that $(n - 1)t_p \leq t$ and substituting

$$[T(t)]_{\text{steady state}} = PR\left[\frac{\varepsilon^{t/\tau}\left(1 - \varepsilon^{-t/\tau}\right)}{1 - \varepsilon^{-t_p/\tau}}\right]$$

$$= PR\left[\frac{\varepsilon^{t/\tau} - 1}{1 - \varepsilon^{-t_p/\tau}}\right] \quad (15\text{-A})$$

$$\lim_{t\to o}[T(t)]_{\text{steady state}} = \lim_{t\to o}\left[PR\left(\frac{\varepsilon^{t/\tau} - 1}{1 - \varepsilon^{-t_p/\tau}}\right)\right]$$

$$= 0 \quad (16\text{-A})$$

To investigate the other limit, we note that $t - t_1 \leq (n - 1)t_p$. Substituting,

$$[T(t)]_{\text{steady state}} = PR\left[\frac{\varepsilon^{(t-t_1)/\tau}\left(1 - \varepsilon^{-t/\tau}\right)}{1 - \varepsilon^{-t_p/\tau}}\right] \quad (17\text{-A})$$

$$\lim_{t\to t_1}[T(t)]_{\text{steady state}} = \lim_{t\to t_1}\left\{PR\left[\frac{\varepsilon^{(t-t_1)/\tau}\left(1 - \varepsilon^{-t/\tau}\right)}{1 - \varepsilon^{-t_p/\tau}}\right]\right\}$$

$$= PR\left(\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t_p/\tau}}\right) \quad (18\text{-A})$$

b. Now, for the case where $(n - 1)t_p + t_1 \leq t \leq nt_p$

$$T(t) = PR\left[\varepsilon^{-(t-t_1)/\tau}\left(1 - \varepsilon^{-t_1/\tau}\right)\left(\frac{\varepsilon^{nt_p/\tau} - 1}{\varepsilon^{t_p/\tau} - 1}\right)\right] \quad (11\text{-A})$$

Rearranging terms,

$$[T(t)]_{\text{transient}} = PR\left[\varepsilon^{-t/\tau}\left(\frac{1 - \varepsilon^{t_1/\tau}}{\varepsilon^{t_p/\tau} - 1}\right)\right] \quad (19\text{-A})$$

The transient portion dies down as t increases indefinitely since,

$$\lim_{t\to\infty}[T(t)]_{\text{transient}} = \lim_{t\to\infty}\left\{PR\left[\varepsilon^{-t/\tau}\left(\frac{1 - \varepsilon^{t_1/\tau}}{\varepsilon^{t_p/\tau} - 1}\right)\right]\right\}$$

$$= 0 \quad (20\text{-A})$$

The steady-state solution is,

$$[T(t)]_{\text{steady state}} = PR\left[\varepsilon^{-(t-nt_p)/\tau}\left(\frac{\varepsilon^{t_1/\tau} - 1}{\varepsilon^{t_p/\tau} - 1}\right)\right] \quad (21\text{-A})$$

As n becomes large, t also becomes large and $(t - nt_p)$ remains finite. Therefore, we need only consider the first period $(n = 1)$.

$$[T(t)]_{\text{steady state}} = PR\left[\varepsilon^{-(t-t_p)/\tau}\left(\frac{\varepsilon^{t_1/\tau} - 1}{\varepsilon^{t_p/\tau} - 1}\right)\right]$$

$$= PR\left[\varepsilon^{-t/\tau}\left(\frac{\varepsilon^{t_1/\tau} - 1}{1 - \varepsilon^{-t_p/\tau}}\right)\right] \quad (22\text{-A})$$

$$\lim_{t\to t_1}[T(t)]_{\text{steady state}} = \lim_{t\to t_1}\left\{PR\left[\varepsilon^{-t/\tau}\left(\frac{\varepsilon^{t_1/\tau} - 1}{1 - \varepsilon^{-t_p/\tau}}\right)\right]\right\}$$

$$= PR\left(\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t_p/\tau}}\right) \quad (23\text{-A})$$

which agrees with equation (18-A).

$$\lim_{t\to t_p}[T(t)]_{\text{steady state}} = \lim_{t\to t_p}\left\{PR\left[\varepsilon^{-t/\tau}\left(\frac{\varepsilon^{t_1/\tau} - 1}{1 - \varepsilon^{-t_p/\tau}}\right)\right]\right\}$$

$$= PR\left[\varepsilon^{-t_p/\tau}\left(\frac{\varepsilon^{t_1/\tau} - 1}{1 - \varepsilon^{-t_p/\tau}}\right)\right]$$

$$= PR\left[\left(\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t_p/\tau}}\right)\left(\varepsilon^{-(t_p - t_1)/\tau}\right)\right]$$

(24-A)

It is obvious that T(max) must occur at time t_1,

$$T(max) = PR\left(\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t_p/\tau}}\right) \qquad (25\text{-A})$$

and T(min) must occur at time t_p,

$$T(min) = PR\left(\frac{1 - \varepsilon^{-t_1/\tau}}{1 - \varepsilon^{-t_p/\tau}}\right)\left(\varepsilon^{-(t_p - t_1)/\tau}\right)$$

$$= T(max)\left[\varepsilon^{-(t_p - t_1)/\tau}\right] \qquad (26\text{-A})$$

We can now obtain the total, or junction, temperature response by summing over all networks in the thermal equivalent circuit.

$$T_j(max) = P\sum_{n=1}^{m} T_n(max) = P\sum_{n=1}^{m} R_n\left(\frac{1 - \varepsilon^{-t_1/\tau_n}}{1 - \varepsilon^{-t_p/\tau_n}}\right)$$

(27-A)

$$T_j(min) = P\sum_{n=1}^{m} T_n(min)$$

$$= P\sum_{n=1}^{m} R_n\left[\left(\frac{1 - \varepsilon^{-t_1/\tau_n}}{1 - \varepsilon^{-t_p/\tau_n}}\right)\left(\varepsilon^{-(t_p - t_1)/\tau_n}\right)\right]$$

(28-A)

A similar analysis can be conducted for other waveforms.

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